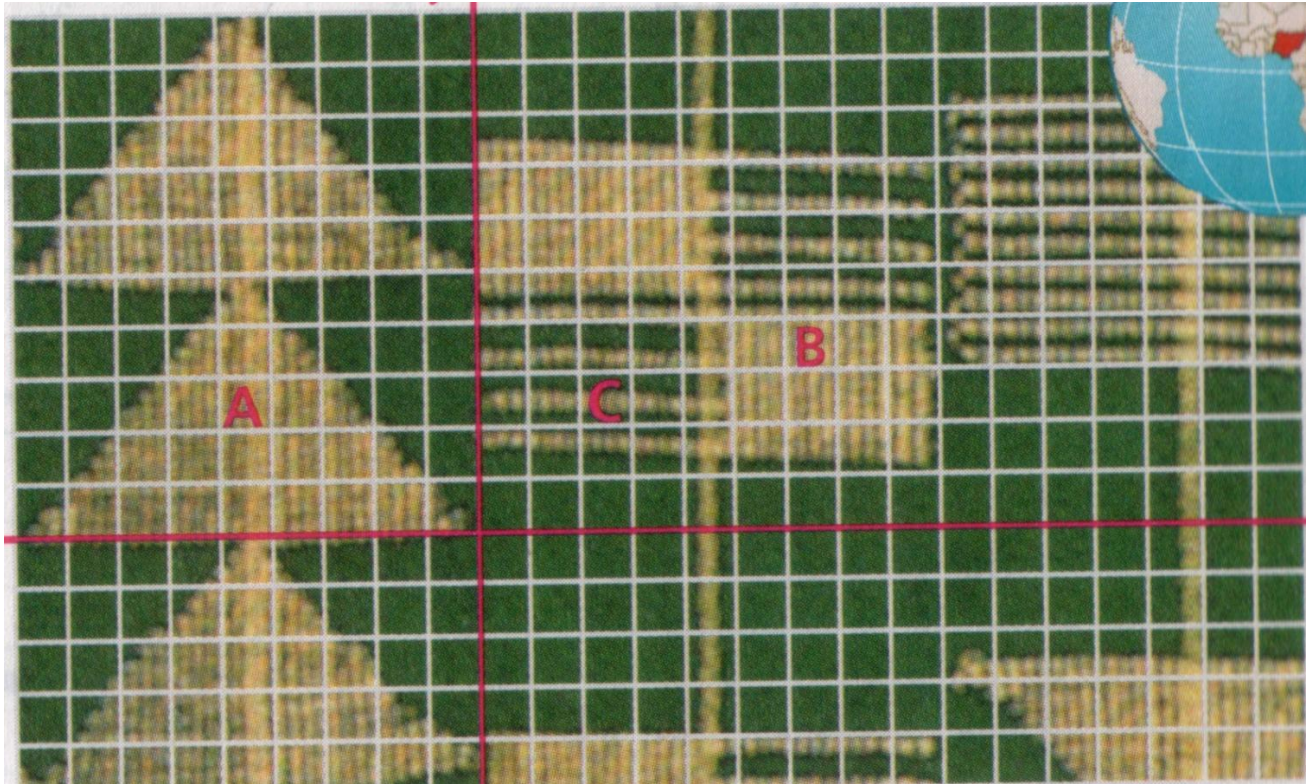


**Introductory Activity:**



You are looking at a textile fabric pattern. Please answer the following questions.

- 1.) What different patterns do you see inside the fabric?
- 2.) How are the shapes in the pattern repeated? Do they shift? Are there any terms for it?
- 3.) Do you recognize any equations in any shapes in the pattern? Try, but it's ok if you don't.
- 4.) Is there anything else unique about the pattern?

## **Absolute Value Transformations**

### **Grade level and Subject:**

College Prep (9-11) Algebra 2 (2 40 minute class periods)

### **Processes Addressed:**

Using technology, making connections, observing, communication, reasoning, visualization, applying.

### **Concepts Addressed/Related Standards:**

#### Concepts

Graphing transformations, reading graphs to determine transformations, absolute value, absolute value transformations, vertical and horizontal translations, stretching and shrinking

#### PDE Content Standards

2.8.11.K Select, justify, and apply an appropriate technique to graph a linear function in 2 variables, including slope-intercept, x- and y- intercepts, graphing by transformations and use of a graphing calculator.

2.8.11.L Write the equation of a line when given the graph of the line, two points on the line, or the slope of the line and a point on the line.

2.8.11.S Analyze properties and relationships of functions.

#### NCTM Standards

1. Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;
2. Recognize and apply mathematics in contexts outside of mathematics.

### **Objectives:**

1. Students will use the Grapher program to explore the graphs of absolute value functions and develop the rules for transforming absolute value graphs.
2. Students will apply the rules for transforming absolute value graphs to write absolute value equations from pictures of graphs and descriptions.
3. Students will apply transformations of absolute value graphs to a real life application.

### **What do I want my students to discover?**

Students will discover the relationships of  $a$ ,  $h$ , and  $k$  in absolute value equations of the form  $y = a|x - h| + k$ . Students should see that  $a$  changes the width and direction of the graph,  $h$  shifts the graph horizontally, and  $k$  shifts the graph vertically. Collectively, these changes are called transformations.

### **Description of Introductory Activity and Discussion:**

1. Students will be presented with patterns that can be represented with transformations. Students will discuss this in their groups.
  - a. What is the pattern made up of?
  - b. How is the pattern repeated?
  - c. Is there anything unique about the pattern?

- d. Is there some kind of equation or function contained in the pattern?
2. Teacher will discuss with the class the pattern and questions in #1.

**Materials Needed:**

1. Laptop cart with Mac laptops and Grapher Program
2. Exploring graphs of absolute value worksheet
3. Textbook

**Description of Activities:**

1. Teacher will give quick overview/demo of Grapher program using the function  $y = |x|$  and several other functions?
2. Students will complete parts 1 through 5 of the worksheet for exploring absolute value graphs. This should be done in pairs, but each student will have a laptop and a worksheet and both will complete the exercise.
3. The entire class will discuss the results and determine the rules for translating absolute value functions. \*\*This will likely conclude day 1 of this lesson.
4. Students will again work in pairs to complete parts 6 and 7 of the worksheet where they will apply what they learned in parts 1 through 5, both in a mathematical context and real-life application.
5. The entire class will discuss the results.

**Typical Discussion Questions:**

1. In the function  $y = a|x - h| + k$ :
  - a. What does  $a$  (positive and negative,  $>1$ ,  $<1$ ) do to the graph?
  - b. What does  $h$  (positive and negative) do to the graph?
  - c. What does  $k$  (positive and negative) do to the graph?
2. Given an absolute value graph, how do I find  $a$ ,  $h$ , and  $k$ ?
3. What is it called when you “move” an absolute value graph with  $a$ ,  $h$ , and/or  $k$ ?

**How students will be encouraged to investigate on their own in the classroom:**

1. Part 7 of the worksheet will be of this variety.
2. Where do you see absolute value or other transformations in real life?
3. How does it apply to our original introductory activity?

**Expected conclusions:**

Students will discover the relationships of  $a$ ,  $h$ , and  $k$  in absolute value equations of the form  $y = a|x - h| + k$ . Students should see that  $a$  changes the width and direction of the graph,  $h$  shifts the graph horizontally, and  $k$  shifts the graph vertically. Collectively, these changes are called transformations.

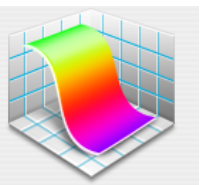
**Assessment:**

See Activities section #3, 5 for in class assessment

HW: pp. 95 – 97 # (Day 1) 2,5,8,9,11,14,16,17,20, (Day 2) 24-28, 38-41, 44,47,48,52,53,59,60

**Applications to real life situations:**

It is difficult to find concrete examples of absolute value transformations in real life, but software programming is a good example. When a software program needs to render patterns on a screen, the program behind it likely uses absolute value transformations (or transformations of other mathematical functions) in its algorithms. Another example of such a software program is one that displays graphs or charts. See if the students can identify some software programs or come up with a small program that uses one (some students might have a programming background or want to try this for extra credit).



## Exploring Graph of Absolute Value with Grapher Program

### Part 1.

1. Graph the equation of  $y = |x|$ .
2. Graph the following equations. Describe how each of the equations are different from the graph of  $y = |x|$ . Be detailed in your description.
  - a.  $y = |x| + 2$
  - b.  $y = |x| + 5$
  - c.  $y = |x| - 3$
  - d.  $y = |x| - 4$
3. In the equation  $y = a|x - h| + k$ , describe what  $k$  will do in the problem. Think about when  $k$  is positive, and when  $k$  is negative.

### Part 2.

4. Graph the equation of  $y = |x|$ .
5. Graph the following equations. Describe how each of the equations are different from the graph of  $y = |x|$ . Be detailed in your description.
  - a.  $y = |x - 2|$
  - b.  $y = |x - 4|$
  - c.  $y = |x + 3|$

d.  $y = |x + 5|$

6. In the equation  $y = a|x - h| + k$ , describe what  $h$  will do in the problem. Think about when  $h$  is positive, and when  $h$  is negative.

Part 3.

7. Graph the equation of  $y = |x|$ .
8. Graph the following equations. Describe how each of the equations are different from the graph of  $y = |x|$ . Be detailed in your description.

a.  $y = 2|x|$

b.  $y = 4|x|$

c.  $y = \frac{5}{2}|x|$

d.  $y = \frac{1}{4}|x|$

e.  $y = \frac{1}{8}|x|$

9. In the equation  $y = a|x - h| + k$ , describe what  $a$  will do in the problem. Think about when  $a$  is greater than one, and when  $a$  is less than one.

Part 4.

10. Graph the equation of  $y = |x|$ .

11. Graph the following equations. Describe how each of the equations are different from the graph of  $y = |x|$ . Be detailed in your description.

a.  $y = 4|x|$

b.  $y = -4|x|$

c.  $y = \frac{5}{2}|x|$

d.  $y = -\frac{5}{2}|x|$

e.  $y = \frac{1}{8}|x|$

f.  $y = -\frac{1}{8}|x|$

12. In the equation  $y = a|x - h| + k$ , describe what  $a$  will do in the problem. Think about the sign of  $a$ .

## Part 5- The Rules.

Fill in the following Rules about the translation of Absolute Value Graphs. Use the word bank to complete the rules about translations.

### Word Bank:

down	horizontally	narrower	up	vertically
down	left	right	up	wider

$$y = a|x - h| + k$$

k shifts the graph of the absolute value \_\_\_\_\_ .

If k is positive, the graph moves \_\_\_\_\_ k units.

If k is negative, the graph moves \_\_\_\_\_ k units.

h shifts the graph of the absolute value \_\_\_\_\_ .

If h is positive, the graph moves \_\_\_\_\_ h units.

If h is negative, the graph moves \_\_\_\_\_ h units.

When a is positive, the graph of the absolute value opens \_\_\_\_\_ .

When a is negative, the graph of the absolute value opens \_\_\_\_\_ .

When a is greater than one, the graph of the absolute value is \_\_\_\_\_ .

When a is greater than zero and less than one, the graph is \_\_\_\_\_ .



## Part 6- Application

1. For each of the following Absolute Value equations, describe how they are different from the graph of  $y = |x|$ .

a.  $y = |x - 5| + 4$

b.  $y = -3|x + 2| - 3$

c.  $y = \frac{1}{5}|x + 3| + 8$

d.  $y = -|x - 6| + 7$

e.  $y = -\frac{1}{2}|x - 1| - 6$

2. Write the equation of the absolute value graph that has the following properties, based on the graph of  $y = |x|$ :

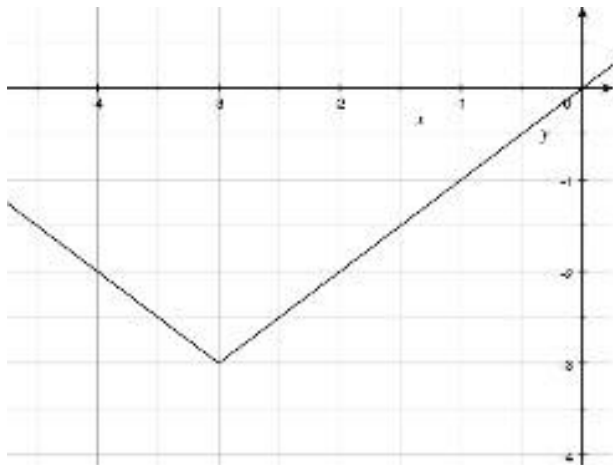
a. Opens up, shifted left 4, shifted down 2

b. Opens down, wider, shifted right 3, shifted down 9

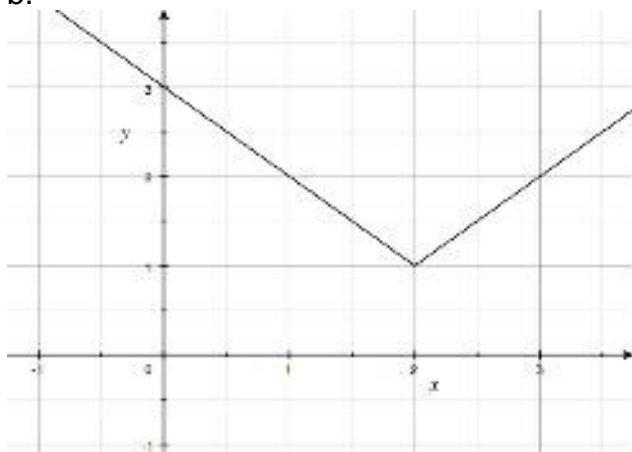
c. Opens up, narrower, shifted left 9, shifted up 4

3. Write the equation of the following graph in the form  $y = a|x - h| + k$ .  
 Note:  $a$  is either positive or negative 1, 2 or  $\frac{1}{2}$ .

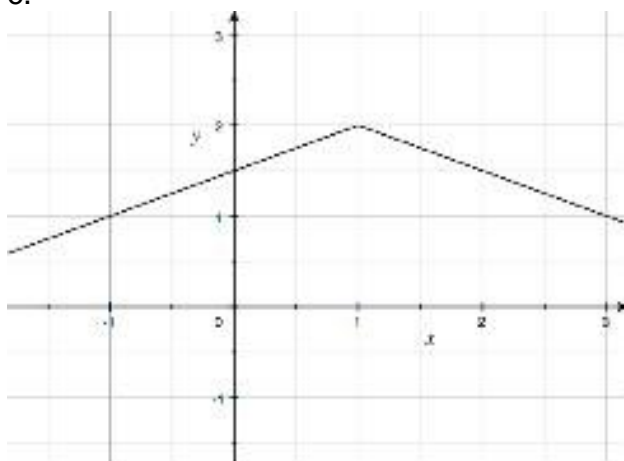
a.



b.



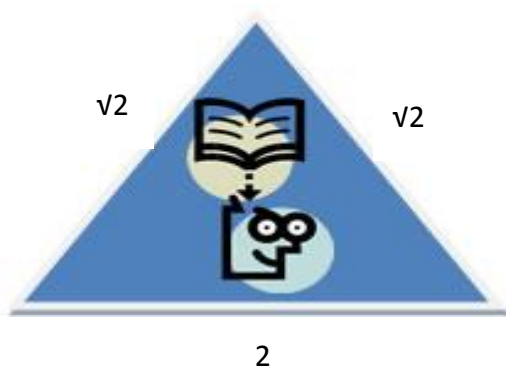
c.



## Part 7 – Making Connections

Suppose you are an awesome graphic designer who has come up with a brand logo that is to be used on posters, floor or wall tile, t-shirts, etc. Your logo is in the shape of an triangle with a cool design in the middle of it. You need to come up with a formula to make sure your logo can be centered on any size square. Examine the logo and answer the questions below to determine the formula.

- a. Here is your logo. Determine the absolute value function that represents the outer triangle based its dimensions. (Note the figure is not quite to scale. **Hint: If it helps, imagine that your logo is on a coordinate plane and the top vertex lies at point (0,0).**)



- b. Suppose you have a square that is 5 units by 5 units and you need to center this logo in it. How can we find the absolute value transformation for centering the logo in this square?
1. Sketch a picture of what the centered logo would look like. Do you notice anything about the center of the grid versus where the top of the logo is?
  2. If the vertex of your logo started at the bottom left of the square, how far would you have to shift it horizontally to center it? Why?

3. Now that you've moved your logo horizontally, how far do you need to move it vertically? Why? (**Hint: You will need to find the height of the triangle.**)
4. Determine the absolute value equation from the information above. Justify.
- c. Suppose the size of the square changed to 6 by 6, what is your absolute value equation?
- d. Can you determine a general equation for the absolute value equation for your logo? Let 'S' represent the length of the sides of the square. Justify.

e. Determine the absolute value transformation if your square is 5 by 5 and your logo size has increased by 50%.

f. Can you determine a general equation for the absolute value transformation with 'S' representing the length of the sides of the square and 'H' representing the height of the triangle? Justify.